

# LINEAR PROGRAMMING TO SELECT AERATION DEVICES FOR FISH FARMS

Awady<sup>1</sup>, M. N., M. A.I.Genaidy<sup>2</sup>, S. A. Ali<sup>3</sup>, M. Abd EL-Backy<sup>4</sup> and W. M. AL-Shorbagy<sup>5</sup>

## Abstract

Linear programming is used to select systems based on minimization of total annual cost using aeration devices for pond to give optimum oxygen. By linear programming model results, one aerator of the type air-injector (0.75 kW) was found suitable for a pond of sizes 0.1, 0.2, and 0.4 ha by least-cost. Splash aerator (0.75 kW) was found suitable for single use for pond of sizes 0.8 ha by least-cost, and suitable in addition to diesel paddlewheel (9 kW) for large pond size (3.2 ha). Diesel paddlewheel aerators (6 kW and 9 kW) were found suitable for large pond sizes (1.6, 3.2, 4.0, and 4.8 ha). Diesel paddlewheel aerators are suitable for large ponds under Egyptian conditions.

**Keywords:** *Linear programming, Aeration devices, Minimization of aeration expenses, Fish pond sizes.*

## INTRODUCTION

Linear programming (LP) is used in aquatic culture in which many of the important water quality parameters are the dissolved gases, such as oxygen, carbon dioxide, hydrogen sulfide, ammonia, and nitrogen. **Forsberg (1996)** developed a new approach to the production planning problem in fish farms. Production planning problems include: the determination of the optimal number of fry to transfer into the grow-out system, the estimation of population growth and production costs, and the determination of the optimal

---

(1) and (2) resp. Prof. Emerit. and Assoc. Prof., Ag. Eng. Ain Shams U. (3) Prof., Ag. Eng. Banha U., (4) Assoc. Prof., Animal Production, Ain Shams U. (5) Res.ASRT..

harvesting schedule in order to maximize profits from the operation. **AL-Duwais and Hebicha (1998)** used linear programming to gain least cost feed of fish feed. Data needed for the application of linear programming method to least cost feed formulation include: (1) nutrient requirements of the animal, (2) limits placed on the level of certain feed ingredients, (3) cost of feed ingredients, and (4) nutrient and energy content of feed ingredients. Aeration, or the addition of dissolved oxygen (DO), is one of the processes most commonly used in aquaculture. Aerator design is based on the amount of oxygen needed, and the minimum Dissolved Oxygen (DO) concentration (**Huguenin and Colt, 1989**). Small paddlewheels are expensive than other electric aerators of similar size because the gear motors used in small paddlewheels are very expensive. For this reason vertical pumps, propeller aspirators, and diffused-air aerators are better suited for small ponds than paddlewheels even though the latter is more efficient to operate (**Boyd, 1990**). Most aerators used in commercial fish ponds induce about 13-23 kg of oxygen per hour, under optimal conditions. Aerated ponds are usually stocked at 1500 fish/ha, and un aerated ponds were are usually at 10000 fish/ha. Maximum daily feeding rates of 60 kg/ha were used in un aerated ponds versus 100-130 kg/ha in aerated ponds, (**Lawson, 1995**).

## **MATERIALS AND METHODS**

**Microsoft Office Excel (2007)** was used to solve the linear programming models and to get the required results.

Linear programming model was developed to select optimal aeration systems based on minimization of total annual cost.

### **Selected aeration devices:**

In the program model, nine aerators, available in Egypt, are used to select from, (seven electrical aerators and two diesel-powered).

Electrical aerators are: (Splash aerator (0.75 kW), Force 7.2 aerator (1.1 kW), New Brio aerator (0.75 kW), Paddlewheel aerator (0.75 kW), Paddlewheel aerator (1.5 kW), Air-injector aerator (0.75 kW), and Air-injector aerator (1.5 kW)), and diesel-powered (Paddlewheel aerator (6 kW), and Paddlewheel aerator (9 kW)).

Determining factor used in this process is kilograms of D.O. per night for aerators as the output scale from aeration.

Table (1) Shows designations for the nine selected aeration devices.

Table (1): Designations of selected aerators.

Aerator	Symbol
Splash (0.75 kW)	X <sub>1</sub>
Force 7.2 (1.1 kW)	X <sub>2</sub>
New Brio (0.75 kW)	X <sub>3</sub>
Paddlewheel (0.75 kW)	X <sub>4</sub>
Paddlewheel (1.5 kW)	X <sub>5</sub>
Paddlewheel Multi-Impeller (6 kW)	X <sub>6</sub>
Paddlewheel Multi-Impeller (9 kW)	X <sub>7</sub>
Air-Injector (0.75 kW)	X <sub>8</sub>
Air-Injector (1.5 kW)	X <sub>9</sub>

**Calculations of total cost for aerators:**

Equation used for calculation of the hourly cost is according to (Awady, 1978), as following:

$$C = P/h (1/a + i/2 + t + r) + (W.e) + m/144$$

Hourly cost for diesel-powered devices is:

$$C = P/h (1/a + i/2 + t + r) + (0.9W.S.F) + m/144$$

Where:

C= hourly cost, L.E/h.

P= price of machine, L.E,

h= yearly working hours, h / year,

a= life expected of machine, year,

i= interest rate / year,

t= taxes, over heads ratio,

r= repairs and maintenance ratio,

0.9= factor accounting for lubrication,

W= power, kW for electrical power.

e= electricity price, L.E/kW.h.

S= specific fuel consumption (L/ kW.h),

F= fuel price, L.E. / L,

m = operator monthly salary, (L.E),

144 = the monthly average working hours,

m/144= monthly wage ratio, L.E,

Finally, linear programming models were used to select the most cost efficient aerator for different pond sizes, and different amounts of aeration.

From **Sadek (2013)** in Egypt and from other manufacturers, capital prices for the available aeration devices were obtained. Price list, design specifications, composition of component parts and photographs were obtained of individual aeration devices.

Useful life of all aerators was assumed 10 years. Interest rate in Egypt is taken 6.2%, taxes, over heads ratio is 25%, and repair and maintenance ratio was suggested 20% by investors and professors.

All calculations for devices were to give the total cost of aeration per hour or one aeration night (10 hours).

## **RESULTS AND DISCUSSION**

### **The linear programming model for selection of aerators:**

The linear programming was used to select optimal aeration systems based on minimization of total annual cost for pond size to give optimum oxygen and intensive culture.

### **Objective function:**

The total cost of aerators per one operating night (10 hours) is calculated by Awady equation for all aerators. The objective function includes the cost per selected aerator multiplied by number of devices ( $x_n$ ) with the results referring to the combination of optimum choice.

$$C = 48x_1 + 50x_2 + 48x_3 + 45x_4 + 50x_5 + 84x_6 + 100x_7 + 44.3x_8 + 54.6x_9$$

Where C = the minimum cost of aeration.

### **Example to calculate the total cost of one aerator which is Paddlewheel (1.5 kW) ( $x_5$ ):**

$$C = P/h (1/a + i/2 + t + r) + (W.e) + m/144$$

$$C = 4444/3650 (1/10 + 0.062/2 + 0.25 + 0.2) + (1.5 * 0.6) + 500/144$$

$$C = 5 \text{ L.E/h.}$$

$$C = 50 \text{ L.E/night.}$$

### **The constraint functions for various pond sizes:**

The constraint function consists of the Standard Oxygen Transfer Rate (SOTR) of the combination of the selected aerators times the symbol of devices and the total dissolved oxygen that pond needs per night (10 ha) for all devices.

$$36 x_1 + 20 x_2 + 21 x_3 + 22.7 x_4 + 43.9 x_5 + 85x_6 + 105x_7 + 17.1 x_8 + 29.5 x_9 \geq \text{OR}$$

Which is 36, 20, 21, 22.7, 43.9, 85, 105, 17.1, and 29.5 = SOTR for  $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8,$  and  $x_9$  by one night res. and OR = pond Oxygen required by addition 4ppm. The constraints for pond depend on the volume of pond. Depth of water in ponds is assumed one meter. By calculating the volume of water in the pond, and by knowing the amount of dissolved oxygen needed to add per liter of water (ppm), the total dissolved oxygen could be calculated. Considering that this amount is 4ppm. The SOTR is calculated for all aerators per night (10 hours).

**Example illustrating the method of solving linear programming by Microsoft Office Excel 2007:**

Target function: to minimize the aeration cost using numbers:  $z_1, z_2,$  and  $z_3$  of aerators of the different types:

$$3z_1 + 2z_2 + 4.5z_3 = \text{minimum} \dots\dots\dots(1)$$

Where, the values 3, 2, and 4.5 are the relative values of aeration expenses for each device.

The constraints of the system include the following relations:

$$36z_1 + 30z_2 + 40z_3 \geq 16 \dots\dots(2), \quad z_1, z_2, z_3 \geq 0 \dots (3).$$

Where, 36, 30, etc.. are the aeration  $O_2$  productivities of each device, reap., in kg/night. 16 = oxygen requirement for area of 10 ha (kg/night).

**The solution by Excel 2007:**

The filling of cells in the Excel working sheet, we resort to the "solver" subprogram, as, shown in Fig. (1). In this program, the target function is set as minimum, the constraints are set as in corresponding cells in the work sheet. Then, the results of  $z_1$ ,  $z_2$ , and  $z_3$  numbers, that yield the target function are read out.

**Step 1:** Fill in with the data. **Step 2:** Data / Solver, Fig (1).

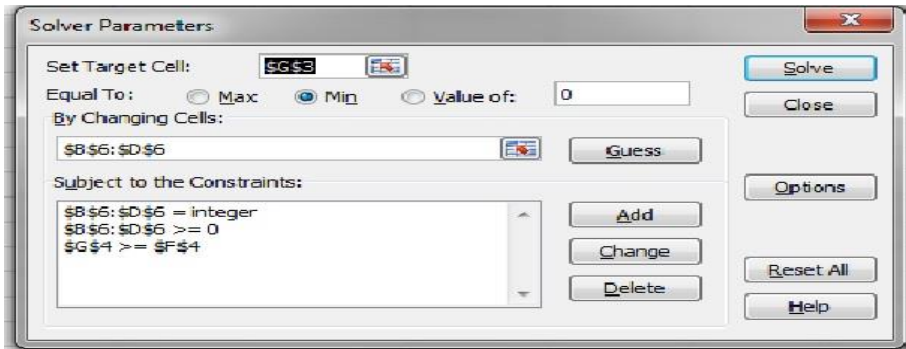


Fig (1), data/ solver.

**Step 3:** The final result show the number of devices used to yield least cost.

From the result we find that  $z_2$  yields 1 device for the selected pond (of 10 ha) 2 L.E with a minimum cost of.

The method of calculation includes least-cost aerators for this pond size to add 4 ppm of dissolved oxygen during 1 night and by calculating the total cost for all aerators and knowing the SOTR for these aerators also. The linear programming model is used after that to calculate the least cost.

The amount of oxygen that must be added to a pond during the night will vary depending on phytoplankton population, feeding rates and biochemical oxygen demand (BOD), among other things. Least-cost aeration systems were selected, based on addition of 4 ppm (mg/l) of dissolved oxygen for the pond sizes: from (0.1 to 4.8 ha).

**The results of linear programming to select optimal aeration systems:**

Table (2) shows the results of linear programming to select optimal aeration systems for various pond sizes.

The linear programming solution was by Excel (2007). Objective function was put in horizontal cells, and constraint functions were put also in horizontal cells under objective function. The horizontal cells results of the device numbers were set to zero before solution.

Linear programming rules were used by "Excel Solver" for the data work sheet. By solving observed three types of results, the first is the selected devices, the second is the resulting least-cost, and the third is the SOTR produced by the selected devices.

Table (2): The results of linear programming to select optimal aeration systems for various pond sizes at 4 ppm of oxygen added.

Pond size (ha)	Aerators types	Aerators numbers	Least-cost (L.E)/night
0.1	(X <sub>9</sub> ) Air-Injector (0.75 kW)	1	44.3
0.2	(X <sub>9</sub> ) Air-Injector (0.75 kW)	1	44.3
0.4	(X <sub>9</sub> ) Air-Injector (0.75 kW)	1	44.3
0.8	(X <sub>1</sub> ) Splash (0.75 kW)	1	48
1.6	(X <sub>6</sub> ) Paddlewheel (6 kW)	1	84



3.2	(X <sub>1</sub> ) Splash (0.75 kW)	1	148
	(X <sub>7</sub> ) Paddlewheel (9 kW)	1	
4.0	(X <sub>6</sub> ) Paddlewheel (6 kW)	2	168
4.8	(X <sub>7</sub> ) Paddlewheel (9 kW)	2	200

These results show the least-cost aerators for varying pond sizes to add 4 ppm of DO to one pond during 1 night (10 hours). These results were obtained by solution of linear programming model by Microsoft Excel, 2007.

Diesel paddlewheel aerators are very appropriate for large pond sizes. For this we recommend the paddlewheel aerators for large ponds.

We recommend using linear programming for aeration problems because it is a technique for optimization of a linear objective function, subject to linear equality and inequality constraints. Linear programming determines the way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model. Some requirements are represented as linear equations.

## **SUMMARY AND CONCLUSION**

According to linear programming results, one aerator of the type air-injector (X<sub>9</sub>, 0.75 kW) is needed for a pond of sizes 0.1, 0.2,

and 0.4 ha by least-cost equal to 9.27 L.E. This selection provides 17.1 kg O<sub>2</sub> per night, but pond needs only 4.2, 8.4, and 16.8 kg O<sub>2</sub> respectively.

Splash aerator (X<sub>1</sub>, 0.75 kW) is suitable alone for pond of size 0.8 ha by least-cost equal to 48 L.E, and suitable with paddlewheel aerator for pond of size 3.2 ha by least-cost equal to 148 L.E.

Diesel paddlewheel aerators (6 kW and 9 kW) are used for large pond sizes (1.6, 3.2, 4.0, and 4.8 ha). This refers to diesel paddlewheel aerators suitable for large ponds under Egyptian conditions.

Diesel paddlewheel aerators are very appropriate for large pond sizes. For this we recommend the paddlewheel aerators for large ponds.

The air-injector aerator was selected for smaller pond sizes because capital investment and annual depreciation charges per kg of induced oxygen were significantly lower than for other types.

Operating cost per kg of oxygen induced was also slightly lower for the air-injector models than for other low-power aerators.

We recommend using linear programming for aeration problems because it is a technique for optimization of a linear objective function, subject to linear equality and linear inequality constraints. Linear programming determines the way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model. Some requirements are represented as linear equations.

## **REFERENCES**

- AL-Duwais,A. and Hebicha, H.A., 1998 , linear programming of fish costs for intensive culture systems in Saudi Arabia , J.King Saud Univ., vol. 10, agric. Sci. (1), :3-15, Riyadh.
- Awady, M.N., 1978, Engineering of tractors and agricutral machinery. Text book, Ain Shams Univ., Fac. of Agric.: 161 – 163. (In Arabic).
- Boyd, C. E. 1990, water quality in ponds for Aquaculture, Aburn, AL: Auburn U/ Alabama Ag Exp St.
- Forsberg O. I., 1996, Optimal stocking and harvesting of size-structured farmed fish: A multi-period linear programming approach, Elsevier j, Math and Comp in Simulation 42: 299-305.
- Huguenin, J. E. and J. Colt, 1989, Design and operating guide for Aq seawater systems, Amsterdam: Elsevier.
- Lawson T. B. 1995. Fundamentals of aquacultural engineering. I T P.
- Microsoft Office Excel, 2007.
- Sadek, S. S., 2013, Price list and design specifications of aerators, Aquaculture consultant office, Cairo: 5.

## المخلص العربى

### البرمجة الخطية لإختيار أجهزة التهوية للمزارع السمكية

محمد نبيل العوضى<sup>١</sup> ، محمد عبدالمجيد جنيدى<sup>٢</sup> ، سمير احمد على<sup>٣</sup> ،  
محمد عبدالباقي<sup>٤</sup> ، وليد محمد الشوربجى<sup>٥</sup>

حسبت تكلفة أجهزة التهوية المتاحة في مصر. حيث أن الهدف الرئيسي هو استخدام البرمجة الخطية لتحديد أنظمة التهوية المثالية التي تعطى التهوية المطلوبة بأقل التكاليف الممكنة. حيث يتم معرفة المعدل القياسي لانتقال الاكسجين (SOTR) لكل جهاز ثم يتم حساب نسبة الاكسجين المتاحة من كل جهاز خلال ليلة عمل (١٠ ساعات)؛ ويتم ذلك أيضا عن طريق تحديد كميات الاكسجين المطلوبة في هذه الفترة لكل حجم من أحجام المرارح المطلوب لها تهوية لكي يتم تزويدها بنسبة أكسجين ٤ ملليجرام أكسجين لكل لتر (4ppm).

طبقا لنتائج البرمجة الخطية؛ حاقن الهواء (0.75 kW) (air-injector) مناسب للمزارع التي تتراوح أحجامها ما بين (٠,٢٥ , ١,٠ ها) حيث هذا الجهاز يمد بكمية أكسجين تقدر ١٧,١ كج أكسجين خلال فترة عمله المقدره (١٠ ساعات)؛ بينما تحتاج هذه المزارع كميات أكسجين تتراوح بين (٤,٢ ؛ ١٦,٨ كج أكسجين).

يعتبر مهوى الرذاذ (0.75 kW) (splash aerator) مناسب لمزرعة بحجم ٠,٨ ها؛ وأيضا مناسباً بالإضافة لجهاز البدال بالنسبة لمزرعة بحجم ٣,٢ ها.

بدالات الديزل مناسبة للمزارع بالحجم الكبير حيث استخدام بدال الديزل (٦ kW) تم اختيارها لمزارع بحجم ١,٦ ها؛ ٤,٠ ها. والبدالات (9 kW) تم اختيارها لمزارع بحجم ٣,٢ ها؛ ٤,٨ ها. يستخدم مهوى الرذاذ (0.75 kW) (Splash aerator) بالإضافة الى البدال (1.5 kW) (paddlewheel) وحاقن الهواء (1.5 kW) (air-injector) لتهوية مزرعة بحجم ٤ ها. ويستخدم أيضا مهوى الرذاذ (0.75 kW) بالإضافة الى البدال (1.5 kW) لمزرعة بحجم ٤,٨ ها؛ لهذه النتائج؛ نوصى باستخدام مهوى الرذاذ (0.75 kW) والبدال (1.5 kW) للمزارع كبيرة الحجم التي تتراوح أحجامها بين ١,٦ ؛ ٤,٨ ها. يوصى باستخدام حاقن الهواء (1.5 kW) للمزارع صغيرة الحجم وذلك يرجع لإنخفاض رأس المال الخاص به ومناسبة كميات الاكسجين الخارجة منه لأحجام هذه المزارع. يوصى؛ طبقا لنتائج البرمجة الخطية؛ باستخدام بدالات الديزل (٦ و ٩ kW) للمزارع كبيرة الحجم؛ وهذا يشير الى ان بدالات الديزل مناسبة للمزارع كبيرة الحجم تحت الظروف.

---

(١)؛ (٢) بالترتيب: أستاذ متفرغ وأستاذ مساعد؛ قسم الهندسة الزراعية؛ كلية الزراعة؛ جامعة عين شمس. (٣) أستاذ بقسم الهندسة الزراعية؛ كلية الزراعة؛ جامعة بنها. (٤) أستاذ مساعد بقسم الأنتاج الحيواني؛ كلية الزراعة؛ جامعة عين شمس. (٥) منحة بحثية بأكاديمية البحث العلمى والتكنولوجيا، القاهرة.